

A linear sequence has 4th and 5th terms like this

$-9, -3, 5, 12, 19, 26, 33, 40, \dots$

Find the first 8 terms of the sequence, the 100th term and the n^{th} term.

$$T_{100} = 7 \cdot 100 - 16 = 684$$

$$T_n = 7n - 16$$

One of the terms will be 61,

Which one?

$$7n - 16 = 61 \quad T_n = 7n - 16$$

$n = 11$

page 365 A1-A6

Sequences

1. Linear: The first difference is a constant

1, 2, 3, 4, $fd = 1$

2. Quadratic: Second difference is a constant

Eg

1	4	9	16	25	36
└──┬──┘		└──┬──┘		└──┬──┘	
3	5	7	9	11	13
└──┬──┘		└──┬──┘		└──┬──┘	
2	2	2	2	2	

3. Geometric:

The ratio of one term to its predecessor
is a constant : $4 \div 2 = 2$ $8 \div 4 = 2$
etc

$T_n = 2^n$ 2, 4, 8, 16, 32, ...

Harder Sequences

Generate the first 5 terms of sequences with the rule:

a) $T_n = n^2 - 2n$ $T_n: -1, 0, 3, 8, 15$

b) $T_n = 3n^2 + 4n - 5$ $2, 15, 34, 59, 90\dots$

c) $T_n = 2^{n-1}$ $1, 2, 4, 8, 16$
 $2^0 = 1$

d) $T_n = \frac{10}{n}$ $10, 5, 3.\dot{3}, 2.5, 2 \dots\dots$

Quadratic sequences

Find the n^{th} term for this quadratic sequence:

4, 15, 32, 55, 84.....

n	1	2	3	4	5
	4	15	32	55	84
fd	11	17	23	29	
sd		6	6	6	
$3n^2$	3	12	27	48	75
$2n-1$	1	3	5	7	9
	$T_n = 3n^2 + 2n - 1$				

Make one for your neighbour.

n	1	2	3	4	5
$3n^2$	3	12	27	48	75
$+2n$	2	4	6	8	10
-1	-1	-1	-1	-1	-1
T_n	4	15	32	55	84

page 366

B1, B3, B5, C2, C4, C6, C8, C10

Proofs

Algebraic method:

Generalising using algebra

By using a counter- example:

Any example that contradicts the theorem disproves it.

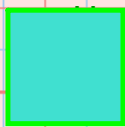
Ex: Prove $2n$ is even for all values of n .

When $n = 2.5$

$2n = 5$ which is odd. ~~∴~~

Ex 369

∴ It is false.

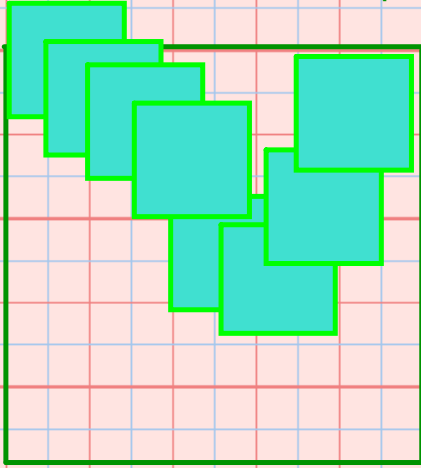


How many 3x3 squares are there in a 10x10 square?

How many 3x3 squares are there in an 11x11 square?

What about in an $n \times n$ square?

Investigate!



In a 10x10 square there are 64 smaller squares

In an 11x11 square there are $(11-2)^2 = 9^2 = 81$

In a $n \times n$ square $(n-2)^2$ small squares

We take away 2 because there are columns at each side where there is no overlapping

In a 10x10 square there will be 49 4x4 squares. You will get 7 down + 7 across

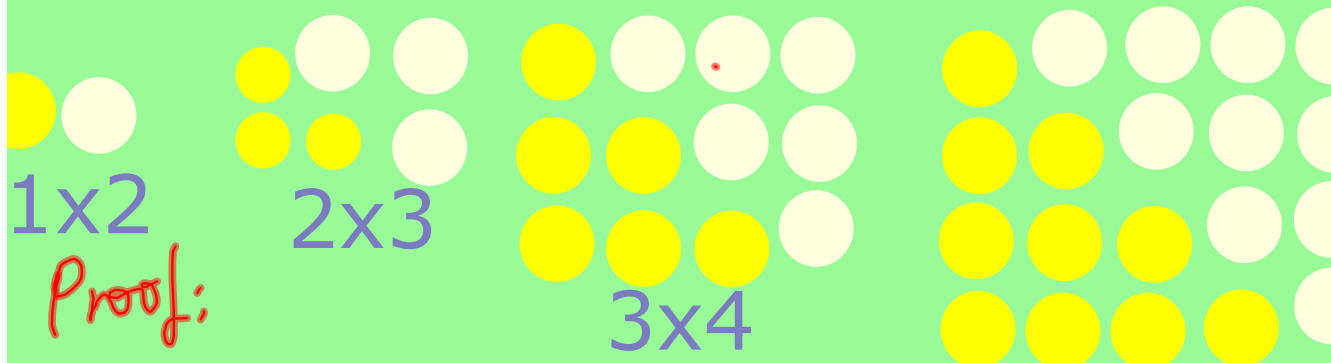
In a $m \times m$ square: $(m-3)^2$ 4x4 squares

Tower of Hanoi



see attachment

A proof of the triangular numbers



Suppose we draw diagrams so that we double the triangular nos making rectangles as shown.

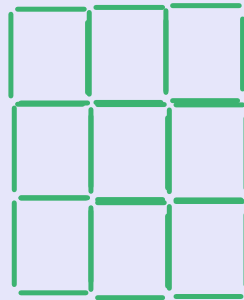
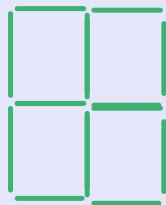
Each rectangle is n high and $(n+1)$ dots wide.

So in each rectangle there are $n(n+1)$ dots.

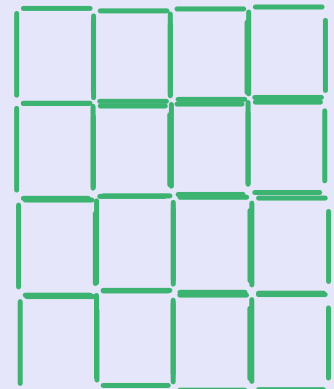
Therefore in the triangle there will be $\frac{1}{2}n(n+1)$ dots. QED

$$\begin{aligned} & (n+2)^2 - n^2 \\ &= (n+2)(n+2) - n^2 \\ &= n^2 + 4n + 4 - n^2 \\ &= 4n + 4 \quad \text{Tur} = 4w + 4 \end{aligned}$$

Finding Rules using structure



1
2
3
4



row
col
 2×1
 $+ 2 \times 1$

$$3 \times 2 + 3 \times 2$$

$$4 \times 3 + 4 \times 3$$

$$5 \times 4 + 5 \times 4$$

$$6 \times 5 + 6 \times 5$$

$$(n+1)n + (n+1)n$$

$$= 2n(n+1)$$

$$= 2n^2 + 2n$$

More algebraic proofs

State whether this is true or false, and give a convincing argument....

The sum of any three consecutive numbers is divisible by 3.

page 372 odd numbers only

Let the first number be n
the second number be $n+1$
the third number be $n+2$

$$\begin{aligned}n + (n+1) + (n+2) &= 3n+3 \\ &= 3(n+1)\end{aligned}$$

Which is a multiple of 3 QED.

b) Let one even no be $2n$
and the other $2m$

$$\begin{aligned}2n - 2m &= 2(n-m) \\ &\text{which is even!} \\ &\text{☺ QED}\end{aligned}$$

$$\begin{aligned}\text{c) } (2m+1)(2x+1) & \\ &= 4mx + 2x + 2m + 1 \\ &= 2(2mx + x + m) + 1 \text{ which is odd QED}\end{aligned}$$

For Tues
p 374 odd nos
only

Attachments

Tower of Hanoi (and beyond).ppt